Short Course on Experimental Dynamic Substructuring

Module #10a: Estimating Fixed-Interface Modes from Measurements on a Flexible Fixture

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Short Course Notes For:
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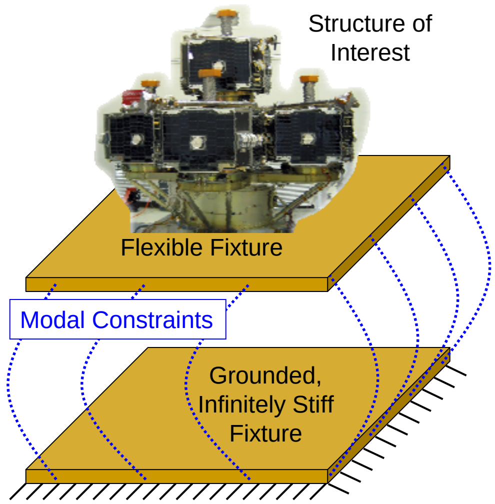
Motivation

- Often tests are required of a structure with a fixed boundary condition, but this usually cannot be adequately realized in the laboratory.
- If measurements are taken on the fixture, substructuring concepts can be used to constrain away its motion and approximate a rigid boundary condition.

Other Applications

- Test campaigns often include:
  - Low level vibration test to obtain modal parameters.
  - Testing on vibration tables at higher amplitudes to test durability.
- It would be preferable to combine these two to reduce cost, test time and risk of damaging the test article.
  - Then one could validate the model for the system using fixed-base modes.



Outline

- Modal Substructuring Theory
  - Modal constraints vs. connection point constraints
- Experiment: Simple plate-beam system
  - Conventional connection point method
  - Modal Constraints
  - Fixture Design
- Experiment: Wind Turbine Blade
- Conclusions

Modal Substructuring Theory

- Equations of Motion for (Fixture+Substructure) found experimentally:

$$\left[\mathbf{I}\right]\{\ddot{\mathbf{q}}\}+\left[{}^{\backslash}2\zeta_r\omega_r{}_{\backslash}\right]\{\dot{\mathbf{q}}\}+\left[{}^{\backslash}\omega_r^{2}{}_{\backslash}\right]\{\mathbf{q}\}=\left[\mathbf{\Phi}_{\mathrm{f}}^{\mathrm{T}}\quad\mathbf{\Phi}_{\mathrm{s}}^{\mathrm{T}}\right]\{\mathbf{F}\}$$

- Modal Parameters of Fixture alone:

$$\mathbf{\Phi}^{\mathrm{fixt}}{}_{\mathrm{f}} \qquad \mathbf{q}^{\mathrm{fixt}}$$

- Fixed-base modes of the substructure are those for which the motion of the fixture is zero.

Physical Constraints	Modal Constraints
$\mathbf{y}_{\mathrm{CPT}}=0$	$\mathbf{q}^{\mathrm{fixt}}=0$

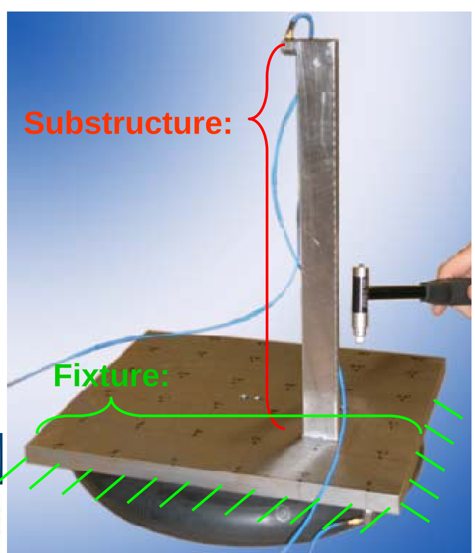
- Use the following constraint equations to make the Fixture motion zero on the (Fixture+Substructure):

$$\mathbf{y}=\mathbf{\Phi q}\approx\mathbf{\Phi}^{\mathrm{fixt}}{}_{f}\mathbf{q}^{\mathrm{fixt}} \qquad \mathbf{y}=\mathbf{\Phi q}\approx\mathbf{\Phi}^{\mathrm{fixt}}{}_{f}\mathbf{q}^{\mathrm{fixt}}$$

Measured

$$\mathbf{\Phi}_{\mathrm{CPT}}\left[\left(\mathbf{\Phi}^{\mathrm{fixt}}{}_{\mathrm{f}}\right)^{+}\mathbf{\Phi}_{\mathrm{f}}\right]\mathbf{q}=0 \qquad \left[\left(\mathbf{\Phi}^{\mathrm{fixt}}{}_{\mathrm{f}}\right)^{+}\mathbf{\Phi}_{\mathrm{f}}\right]\mathbf{q}=0$$

Hardware / Experimental Setup

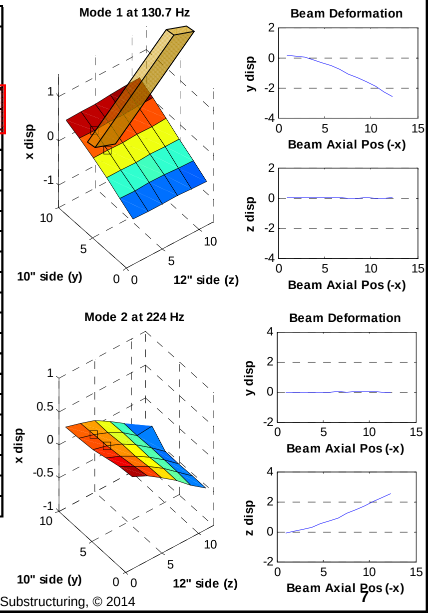


- Objective: Find the fixed-base modes of the beam.
- Experimental Design:
  - 3rd beam nat. freq. (2^{nd} y bending) = 1^{st} plate nat. freq.
  - 6^{th} beam natural freq. (2^{nd} z bending) = 3^{rd} plate nat. freq.
  - The plate (fixture) is not much stiffer than the system of interest (beam); this would not be an optimal fixture design.
- Assures that the modes of the two systems are tightly coupled creating a difficult substructuring problem.

N_o = 48 + 26 = 74

Plate+Beam Modes

Nat. Freqs (Hz) of Elastic Modes				
Mode	Plate Alone (Fixture)	Mode	Plate + Beam	Plate+Beam Modes:
1	670.5	1	130.7	Beam 1y
2	893.7	2	224.0	Beam 1z
3	1344.0	3	628.3	Beam 2y + Plate
4	1620.3	4	693.8	Beam 2y + Plate
5	1850.4	5	902.4	Plate
6	2538.0	6	1254.4	Beam 2z + Plate
7	3107.8	7	1350.2	Beam 2z + Plate
8	3143.6	8	1657.8	Plate
9	3591.1	9	1770.7	Beam 3y + Plate
10	4082.0	10	1797.8	Beam 3y + Plate
11	4814.5	11	1906.1	Beam 3y + Plate
12	4837.7	12	2311.9	Plate
13	5220.6	13	2995.7	Plate
14	5561.4	14	3107.7	Plate
15	6765.1	15	3233.0	Plate
16	7093.4	16	3424.5	Beam 3z + Plate
17	7147.8	17	3522.8	Beam 4y + Plate
18	7179.0	18	3845.5	Plate
19	7808.4			
20	8288.7			
21	8414.8			



Plate+Beam Modes (2)

Nat. Freqs (Hz) of Elastic Modes				
Mode	Plate Alone (Fixture)	Mode	Plate + Beam	Plate+Beam Modes:
1	670.5	1	130.7	Beam 1y
2	893.7	2	224.0	Beam 1z
3	1344.0	3	628.3	Beam 2y + Plate
4	1620.3	4	693.8	Beam 2y + Plate
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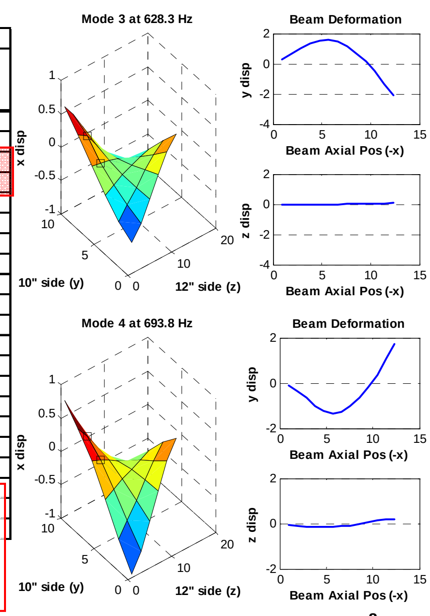
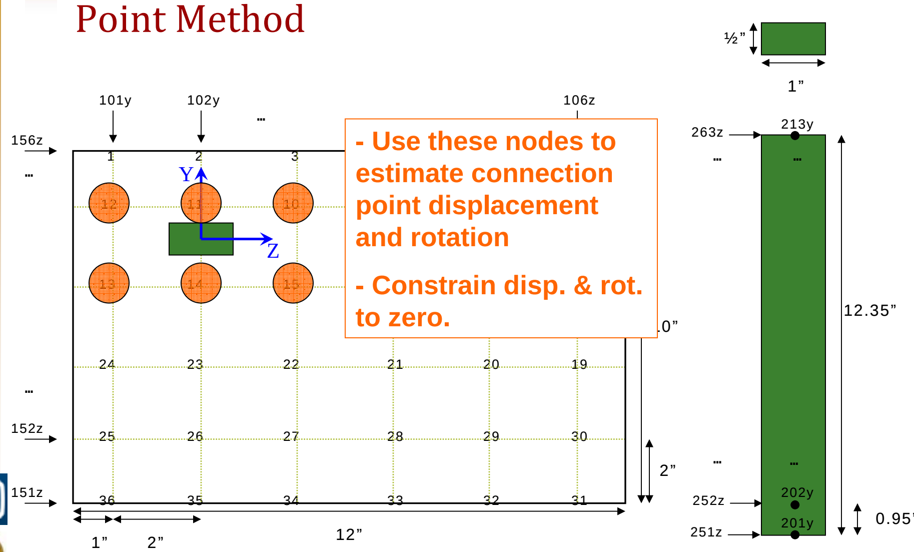
18	7179.0	18	3845.5	Plate
19	7808.4			
20	8288.7			
21	8414.8			

Modes 1-18 used subsequently, plus 6 rigid body modes from an FEA model.



Substructuring with Conventional Connection Point Method

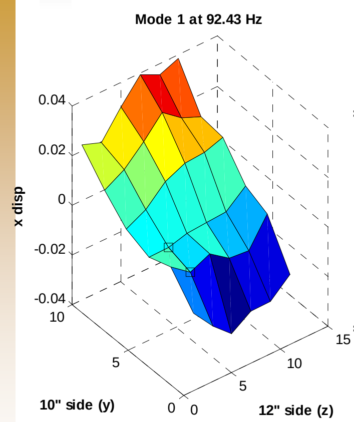


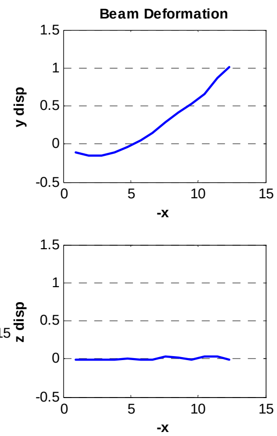
Conventional (CPT) CMS Results

Experimental Plate + Beam Frequency (Hz)	CMS Prediction using CPT Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
130.66	92.4	107.35	-13.9%
224.03	169.6	214.70	-21.0%
628.3	209.4	spurious	-
693.8	347.7	spurious	-
902.4	651.3	672.8	-3.2%
1254.4	665.2	672.8	-1.1%
1350.2	1234.8	1345.5	-8.2%
1657.8	1367.5	1345.5	1.6%
1770.7	1666.7	1883.4	-11.5%
1797.8	1778.8	1883.4	-5.6%
1906.1	1830.7	1883.4	-2.8%
2311.9	2111.5	spurious	-
2995.7	2870.4	spurious	-

- 1st two modes of the fixed-base beam are reasonably well estimated.
- Multiple estimates are obtained for the higher modes.
- Spurious modes remain in the frequency band of interest. These should have been removed by the CMS procedure (due to the constraints).

Conventional (CPT) CMS Mode Shapes





- 1st fixed-base bending mode of the beam (y-dir).
- Residual rotation at the connection point.
  - The plate is not completely stationary, and some of its motion affects the beam due to the imperfect constraints.
- Similar results for other modes.

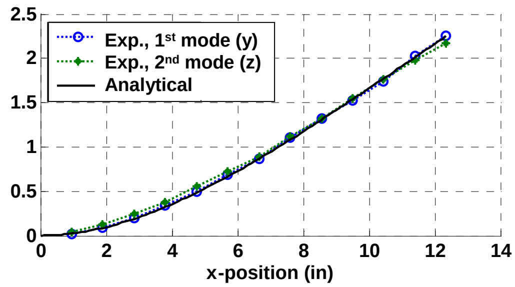
Modal Constraints (12p/24pb)

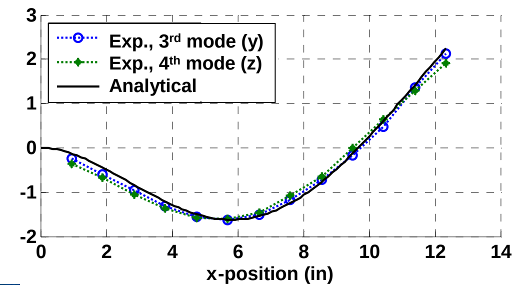
Experimental Plate + Beam Frequency (Hz)	CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
130.66	104.1	107.35	-3.0%
224.03	195.2	214.70	-9.1%
628.3	651.4	672.8	-3.2%
693.8	1230.5	1345.5	-8.6%
902.4	1777.1	1883.4	-5.6%
1254.4	1823.0	3691.4	-50.6%
1350.2	2369.4	3766.8	-37.1%
1657.8	2923.8	6102.2	-52.1%
1770.7	3045.3	7382.9	-58.8%
1797.8	3352.8	9115.6	-63.2%
1906.1	[illegible]	[illegible]	-71.3%
2311.9	3620.2	12731.7	-71.6%
2995.7	-	-	-

Bandwidth of Substructures:
(Plate+Beam) 4 kHz
Plate alone 3 kHz

- CMS results found by coupling 12 experimentally measured plate modes to 24 experimental plate + beam modes.
- Six fixed-base modes of the beam are reasonably estimated.
- Modes with y-direction bending estimated more accurately (less stiff).

Substructuring Results (12p/24pb)

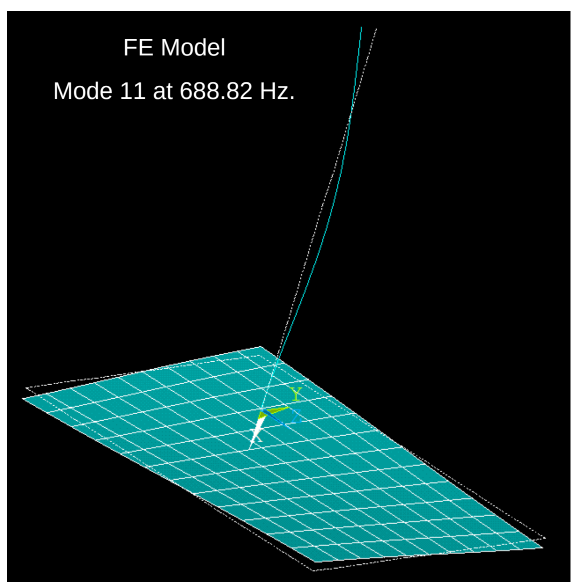




- Mode Shapes of 1st and 2nd y- and z- bending modes
- CMS Prediction using measured modes for plate and plate+beam.
- Both modes predicted very accurately.
  - Small difference in shapes perhaps due to slope near the root of the beam.
  - Why are frequencies not more accurate?

Analytical Results

- A simple finite element model was created and used to compute the CMS results that would be obtained with perfect measurements.
- Plate meshed with 1.0 inch square 8-node shell elements.
- Beam meshed with 2-node beam elements.
- Mode shapes of first 30 modes of plate and plate+beam found and exported to Matlab.



Analytical Substr. Results (12p/24pb)

FEA Plate + Beam Frequency (Hz)	CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
126.34	103.25	107.35	-3.8%
207.2	183.8	214.7	-14.4%
630.96	647.53	672.77	-3.8%
688.82	1180.5	1345.5	-12.3%
899.33	1810.4	1883.4	-3.9%
1207.9	2431	3691.4	-34.1%
1349.2	2930.6	3766.8	-22.2%
1641.3	3072.3	6102.2	-49.7%
1786.1	3345.6	7382.9	-54.7%
1895.3	3389.2	9115.6	-62.8%
2316.9	3536.2	12204	-71.0%
2996.3	4173.8	12732	-67.2%

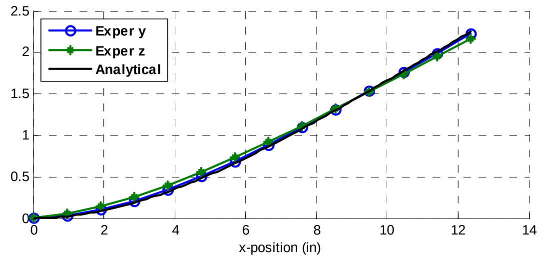
- Results using CMS with FE modes show a similar level of error as obtained experimentally!

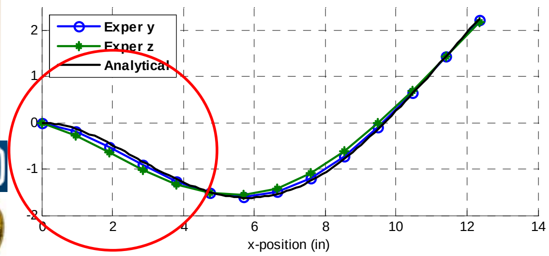
Effect of Number of Modal Constraints

FEA Plate + Beam Frequency (Hz)	CMS Prediction of Fixed-Base Frequencies (Hz) vs. # of Modal Constraints							Analytical Fixed Base
	6	8	10	12	14	16	18	
126.34	102.07	102.65	103.04	103.25	103.27	103.28	107.52	107.35
207.2	175.91	181.84	183.15	183.8	184.45	184.52	202.39	214.7
630.96	623.49	643.32	646.2	647.53	647.63	647.68	679.08	672.77
688.82	683.91	1157.6	1176.6	1180.5	1183.5	1183.8	1267.4	1345.5
899.33	892.38	1244.7	1674.6	1810.4	1810.8	1811	1929.9	1883.4
1207.9	1181.3	1634.8	1825.4	2431	2446.4	3075.1	3421	3691.4
1349.2	1253.2	1689.1	2198.2	2930.6	3326.9	3332.9		3766.8
1641.3	1636.3	1826.3	2802.7	3072.3	3383.5	3535.4		6102.2
1786.1	1690.1	2235.3	2933.3	3345.6	3535.2			7382.9
1895.3	1830.4	2822.3	3072.5	3389.2	4116.6			9115.6
2316.9	2262.5	2943.6	3347.6	3536.2				12204
2996.3	2838.4	3072.6	3425.1	4173.8				12732

- Natural frequencies converge very slowly as the number of modal constraints increases.
  - Exception: with more than 16 constraints too few modes remain in the system to describe its motion, so the results change dramatically.

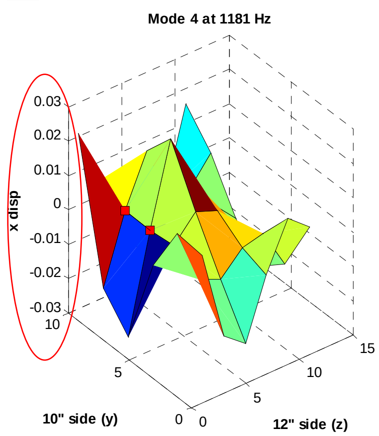
Analytical Substr. Results (12p/24pb)

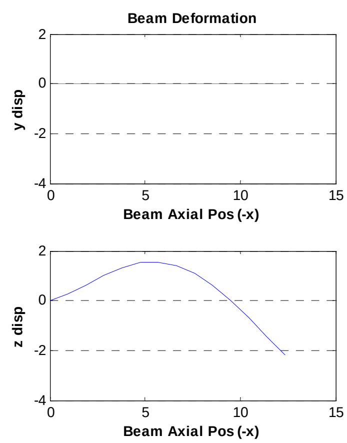




- As with experimental results, the mode shapes agree quite well with the analytical shapes for a cantilever beam.
- But, is there some residual rotation at the base of the beam?

Mode Shapes of Plate+Beam **after Coupling**

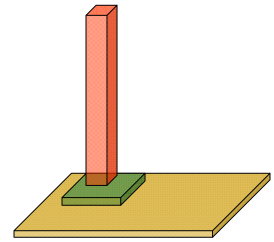




- Some residual rotation is visible at the base of the beam in the bending direction in each of these modes.
  - But, the observed residual rotation is not large enough to cause a significant deflection of the tip of the beam.

CMS with Modified Fixture

FEA Plate + Beam Frequency (Hz)	CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
122.00	107.0	107.35	-0.3%
251.39	211.9	214.70	-1.3%
667.0	669.2	672.8	-0.5%
778.1	1317.3	1345.5	-2.1%
980.9	1867.4	1883.4	-0.8%
1299.1	3211.4	3691.4	-13.0%
1524.4	3539.4	3766.8	-6.0%
1819.7	3638.7	6102.2	-40.4%
1896.4	3652.5	7382.9	-50.5%
2173.6	3768.2	9115.6	-58.7%
2898.0	3904.8	12204.3	-68.0%



- Analysis repeated using FEA model with the section near the base of the beam three times as thick
- Errors reduce significantly!
- **Conclusion: Frequency errors caused by local deformation of the plate that is not captured by the free modes of the plate!**

Alternative: SVD Modes

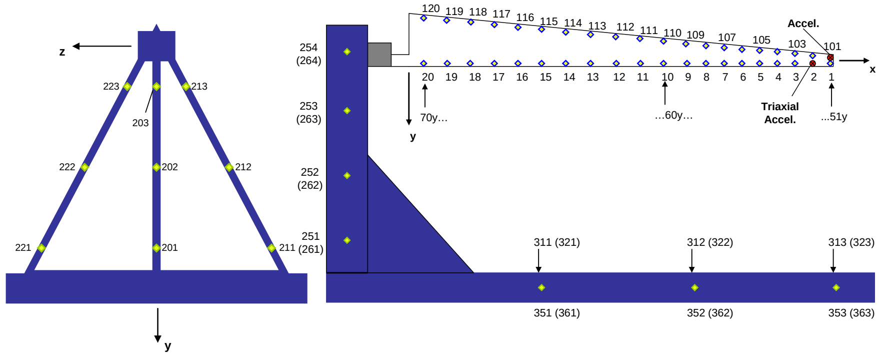
Doesn't require an estimate for the free-modes of the plate!!!

Experimental Plate + Beam Frequency (Hz)	CMS Prediction with 6-RB & 9 SVD Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
126.34	103.3	107.35	-3.8%
207.20	182.2	214.70	-15.1%
631.0	647.6	672.8	-3.7%
688.8	1169.0	1345.5	-13.1%
899.3	1806.8	1883.4	-4.1%
1207.9	1835.3	1883.4	-2.6%
1349.2	2793.2	spurious	-
1641.3	3339.5	3691.4	-9.5%
1786.1	3535.6	3766.8	-6.1%
1895.3	**24 P+B – 15 = 9**	-	-
2316.9		-	-
2996.3		-	-
3119.9	-	-	-

- Constraints:
  $\Phi(:,7:24) = U^*S^*V^T$
  $[\Phi(:,1:6), U(:,1:9)]^+\{y_f\} = 0$
- Results are comparable to those obtained with free-modes of the plate, but again the frequency errors are a concern.
- Again, convergence is very slow as the number of constraints increases.

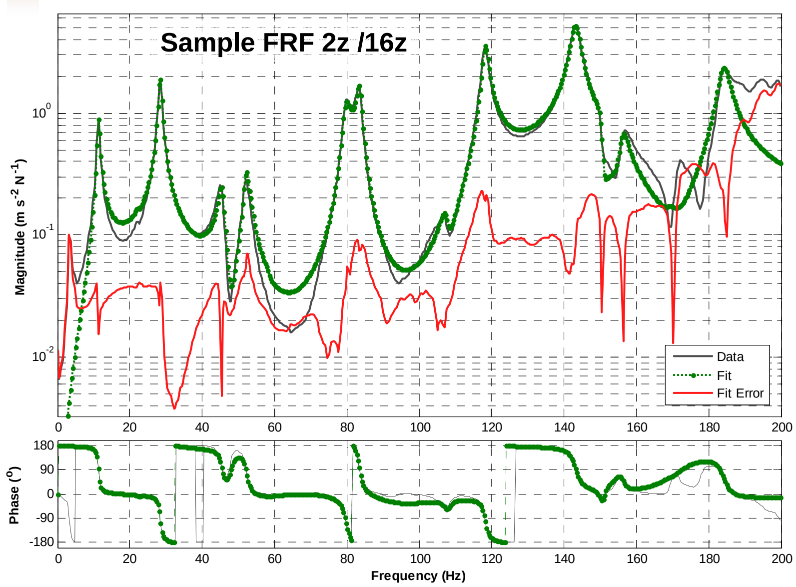
- In some cases one may not have an adequate fixture model. What can be done with the measured shapes only?
- This slide shows the result of using the dominant shapes observed on the fixture (for a test of fixture+structure) to form constraints.
  - The rigid body modes were used specifically to assure that all of the rigid body motion disappeared.

Modal Test on Wind Turbine Blade



- Test performed in a stiff frame that was used for static fatigue testing.
- Desire to correlate modes with a fixed-base model for the blade.

Curve fit performed on to all measurements



Modes After Applying SVD Constraints

Mode Num.	Mode Shape Description	Natural Freq. (Hz)	2 SVD Constraints		3 SVD Constraints	
			f_n CMS	% Diff	f_n CMS	% Diff
1	FW B1	3.36	3.83	12.1%	3.84	12.4%
2	EW B1	5.24	5.27	0.5%	5.28	0.7%
3	FW B2	11.40	11.44	0.4%	11.64	2.1%
4	EW B2	22.42	22.52	0.4%	22.77	1.6%
5	FW B3	28.44	28.85	1.4%	29.54	3.7%
6	FW B4, Fixture+	45.50	48.92	7.0%	50.26	9.5%
7	FW B4, Fixture–	52.26	-	-	-	-
8	EW+FW	53.37	-	-	-	-
9	EW B3	58.29	56.52	-3.1%	56.96	-2.3%
10	1st Torsion	80.01	79.96	-0.1%	79.97	0.0%
11	FW B5	83.54	81.84	-2.1%	83.90	0.4%
12	EW B4	107.37	106.85	-0.5%	107.01	-0.3%
13	FW B6	118.25	115.77	-2.1%	119.75	1.2%
14	2nd Torsion	143.47	143.45	0.0%	143.54	0.0%
15	FW B7, Tors.	150.29	150.12	-0.1%	154.12	2.5%
16	FW B7, Tors.	156.21	154.18	-1.3%	-	-
17	EW B5 +FW	169.61	168.30	-0.8%	159.09	-6.6%
18	FW B7, EW B8, Torsion	184.11	183.02	-0.6%	182.97	-0.6%

- Natural frequencies of wind turbine blade in frame.
- Modes correspond to:
  - Edgewise bending (EW)
  - Flapwise bending (FW)
  - Torsion.

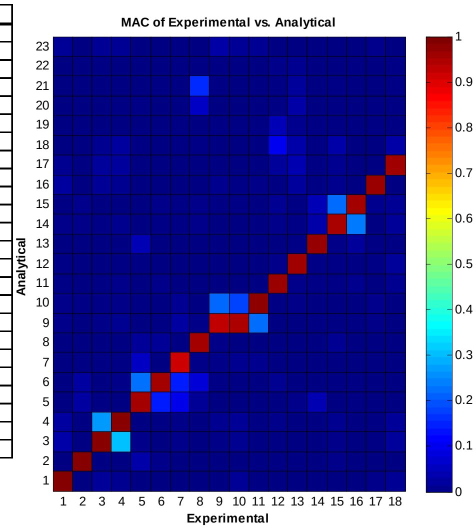
Conclusions

- Objective: Estimate fixed-base modes of a structure from measurements on the structure and a fixture.
- Conventional Approach: Estimate the connection point displacement and rotation and use CMS to force them to zero.
  - Spurious modes were present in the CMS predictions.
  - There were significant errors in many of the natural frequencies.
- Modal Constraints: Estimate the modal motion of the fixture and force it to zero.
  - Obtained virtually identical results using FEA and experimental measurements, and the measurements were acquired using the simplest, cheapest test method => the CMS method is robust!!
  - However, there were relatively large errors in natural frequencies (~5% or more) because the fixture (plate) was soft relative to the substructure. The fixture should be carefully designed to minimize this.
  - CMS converged smoothly as the number of modes increases, and the results were always qualitatively reasonable.

Extra Slides

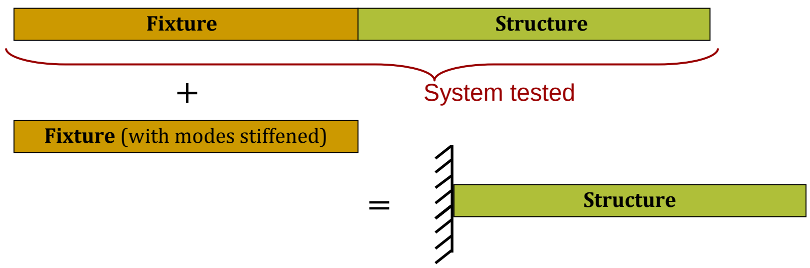
Experimental / Analytical Correlation

Mode	Experiment	FEA Model	MAC	MSF
1	130.66	126.34	0.9966	1.03
2	224.0	207.2	0.9960	1.01
3	628.3	631.0	0.9928	0.98
4	693.8	688.8	0.9942	1.04
5	902.4	899.3	0.9655	1.06
6	1254.4	1207.9	0.9676	0.91
7	1350.2	1349.2	0.9266	1.05
8	1657.8	1641.3	0.9675	1.06
9	1770.7	Torsion?		
10	1797.8	1786.1	0.9586	0.74
11	1906.1	1895.3	0.9893	0.98
12	2311.9	2316.9	0.9765	1.08
13	2995.7	2996.3	0.9713	1.10
14	3107.7	3119.9	0.9800	1.03
15	3233.0	3255.6	0.9617	0.99
16	3424.5	3383.8	0.9681	1.48
17	3522.8	3552.2	0.9752	0.91
18	3845.5	3836.6	0.9704	0.73
19		4455.3		
20		4866.9		
21		5102.3		
22		5395.4		
23		5717.8		
24		5827.2		



- Table compares the natural frequencies of the plate+beam between the experiment and FEA model.

Simple Example - Beam



- First we considered a very simple example, a 12”x0.75”x1” steel beam.
  - A simulated test is performed with this beam coupled to another identical beam as shown.
  - We then use MCFS to estimate the fixed-base modes of the structure (beam on the right) from measurements on the test structure.

Note on constraints

- One way to constrain the fixture to ground:
  - $[\phi_f]^+\{y_f\}= [\phi_f]^+\{y_{f+s}\}$, where
    - $[\phi_f]$ = fixture mode shapes
    - $\{y_f\}$ = measurement points on fixture when it was tested in isolation
    - $\{y_{f+s}\}$ = measurement points on fixture when it was tested with the structure of interest attached
    - $()^+$ = pseudoinverse
  - This projects the motion of the (f+s) system onto each of the fixture modes and constrains the two. The fixture natural frequencies are then made very large to approximate a fixed base.
- Another way to do this is to constrain the projection of the fixture modes to zero (limit of an infinitely stiff fixture).
  - $[\phi_f]^+\{y_{f+s}\} = 0$

Natural Frequencies Before Coupling:

- Presuming that we can measure all of the modes of each system out to 8000 Hz, we obtain 5 and 7 modes with the following frequencies:

Mode	Fixture	F + S
1	0	0
2	0	0
3	1089.3	272.3
4	3002.8	750.7
5	5886.7	1471.7
6		2432.7
7		3634.1

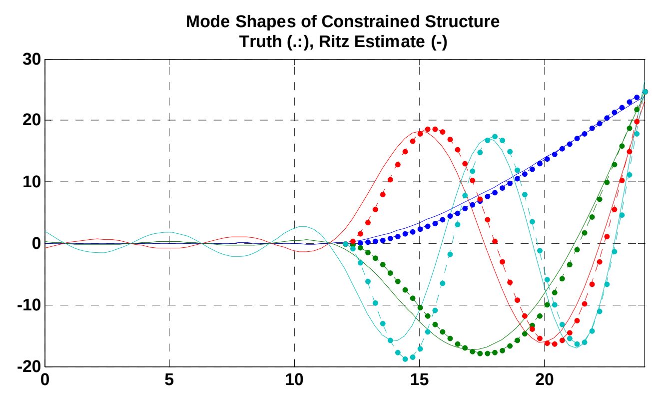
Modes after Coupling (5/7 Modes)

Modes of Fixed-Base Structure

Mode	Actual	CMS	% Error
1	171.2	145.6	-15.0
2	1072.8	924.7	-13.8
3	3004.0	2612.1	-13.0
4	5886.6	5140.8	-12.7

- Natural frequencies are in the ball park, but there are considerable errors.

- Mode shapes are similar to the true ones, but with only 5 constraints the motion of the fixture cannot be completely eliminated.



Modes after Coupling (11/21 Modes)

Modes of Fixed-Base Structure

Mode	Actual	CMS	% Error
1	171.2	161.4	-5.7
2	1072.8	1014.5	-5.4
3	3004.0	2846.9	-5.2
4	5886.6	5590.4	-5.0
5	9731.0	9258.1	-4.9
6	14536.6	13850.9	-4.7
7	20303.7	19367.6	-4.6
8	27032.5	25802.0	-4.6
9	34723.6	33136.7	-4.6
10	43377.7	41317.5	-4.7

- Everything is converging as the number of modes increases, but even with 11 and 21 modes (out to 45,000 Hz) there are 5% errors in the natural frequencies.

- This appears to be the usual issue of free-free modes converging slowly near an interface. Also, this is an extreme example where the fixed base is just as flexible as the structure of interest. The fixed base will usually be more rigid than the structure.